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Equalization Methods with True Response using Discrete Filters

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ABSTRACT

Equalizers with fixed frequency filter bands, although successful, have historically had a combined frequency response that at best only roughly matches the band amplitude settings. This situation is explored in practical terms with regard to equalization methods, filter band interference, and desirable frequency resolution. Fixed band equalizers generally use second-order discrete filters. Equalizer band interference can be better understood by analyzing the complex frequency response of these filters and the characteristics of combining topologies. Response correction methods may avoid additional audio processing by adjusting the existing filter settings in order to optimize the response. A method is described which closely approximates a linear band interaction by varying bandwidth, in order to efficiently correct the response.

1. BACKGROUND

The audio graphic equalizer has evolved into a set of around thirty filters at fixed frequencies, covering the audio range. The operator has adjusted the level of each individually, either to correct a magnitude response variation, or to create one intentionally. This degree of control has remained popular despite the recent advances of technology. Digital Signal Processing (DSP) has made it practical to provide many times this resolution, to the point where the term arbitrary magnitude response is considered applicable.

As is well known, equalization has phase shift as a mathematical requirement. Phase shift has not always been considered as important as magnitude response because it was less audible. Still, studies have confirmed that it is audible in some situations [1,2]. Minimum phase is often chosen for its economy and because it is appropriate for correcting a system with minimum phase characteristics, which may be cancelled by minimum phase filtering without adding time delay [3]. Minimum phase filters might also be appropriate for magnitude correction of system responses with unknown phase characteristics. Today, mainly due to DSP, non-minimum phase shift designs have become more practical, and linear phase with its constant group delay is the most common example.

The graphic equalizer with front panel sliders displaying a response curve may be easily misunderstood as displaying the actual magnitude response. Although discrete filters can be designed with narrow bandwidths, even approaching a filter shape with a flat top and steep cutoff is expensive, so in practice each filter has an effect over a wider range of frequencies than it is intended to affect. The filter response curves still have significant magnitude at neighboring filter band frequencies. In practice, each filter significantly affects no more than the first few neighboring frequency bands. Figure 1 shows the combined response for several topologies. The resulting frequency response then doesn't match the settings (at band center frequencies). The nature of the combined response depends on the filter combining topology.

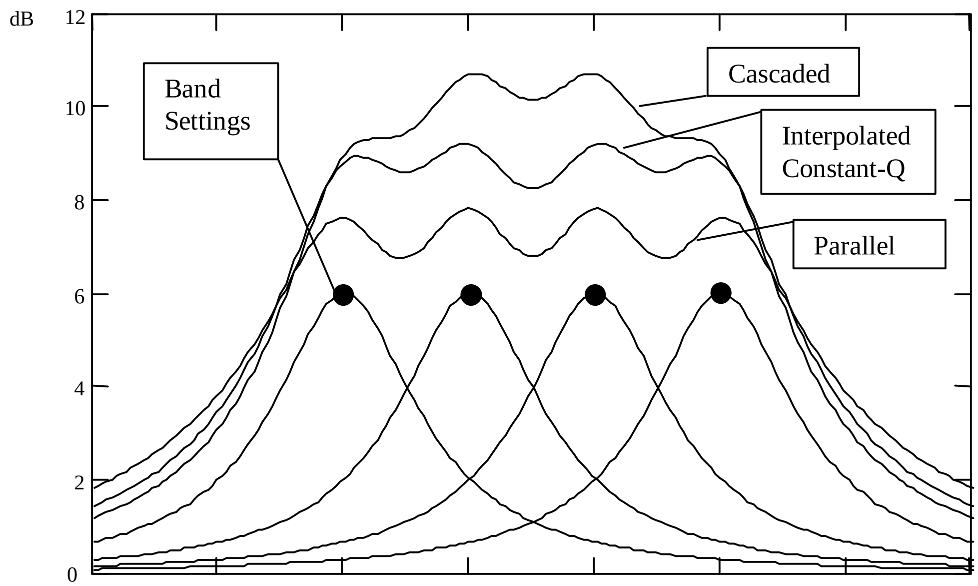


Figure 1: Combined Responses – Settings at +6 dB

Using discrete filters, techniques can be applied to counteract this filter skirt interaction, and produce what is termed "true response", that is, the frequency response closely matches the controls. Each band becomes independent, or very nearly so. Otherwise, to be effective, an operator must be very accustomed to the product's particular filter and combining behavior.

Equalizer filters began as analog second-order filters, and have since been implemented as digital IIR filters. Equalization curves with complex shapes can also be accomplished with single large FIR filters, allowing multiple frequency band settings to be combined into one filter. The number of bands is limited only by the size of the filter. An FIR filter may be designed to approximate the impulse response of many IIR filters, and provides the ultimate in flexibility, where magnitude and phase may be adjusted more specifically and semi-independently. In order to support low frequencies, many thousands of taps are required. In order to economically implement this, complicated methods like multirate processing or the use of FFT for fast circular convolution are used. These methods are outside the scope of this paper with some examples in [5,6,7,8].

Banks of IIR filters have some advantages over single FIR filters, namely simplicity and speed of design, and better efficiency for a limited number of bands that include low design frequencies [4]. Analog filters are designed once, while a digital filter may need to be redesigned when any parameter changes. Small IIR filters can be adjusted (redesigned) quickly, compared to large FIR filters.

Equalizers are usually adjusted manually, but in theory an automated analysis of the sound system may be used. Care then must be taken with correcting nonminimum phase responses, where matching the phase requires added delay. Also, equalization of deep notches in the response should not be attempted because of potential adverse effects in other areas of a room, as well as the possibility of amplifier overload. These techniques are also beyond the scope of this paper.

2. DISCRETE FILTERS AND REQUIRED FREQUENCY RESOLUTION

Because of the sheer quantity of filters, using discrete filters generally limits the practical frequency resolution to one-third octave. How much of a limitation this is is debatable, and the one-third octave equalizer is widely recognized as effective, familiar, and easy to use.

A discrete filter equalizer is the most practical choice for analog designs, given the difficulty of making transversal (FIR) analog filters. Second order discrete filters have been universally used, and their digital versions are designed using equations that can be fairly simple (or sophisticated) [9-14]. These equalizers have proven to be cost-effective, popular, and effective. Note that thirty bands at one-third octave spacing have an effect over roughly ten octaves, covering the widely accepted 20 Hz to 20 kHz range.

The ideal equalizer would have unlimited frequency resolution, but what is really useful? Human hearing has limited resolution, with a physical component defined by the critical bands, and a psychophysical pitch resolution that's around 25 times better [15,16]. The critical bands are around one-sixth octave wide above 1 kHz, increasing beyond one-third octave below this. Their exact characteristics may be a matter of debate. They are not at fixed frequencies but are a physical ability to resolve a combination of frequencies. In the presence of a complex musical signal, critical bands limit our ability to hear narrow response variations, because of masking caused by closely spaced frequency components. Signals with fewer spectral components enable perception of much narrower response variations.

Even assuming unlimited perceptual abilities, there are other practical limitations. A typical room is not anechoic, and has more than one listener position, so it has a response that can't be perfectly flattened. The frequency and phase response vary with listener position because of room reflections and off-axis speaker response variations. The most that can be done is to flatten the speaker/crossover and make some compromise for the room reflections [17,18]. Of course a good sound system won't have too many severe narrow variations.

It appears that one-sixth octave resolution matches the minimum size of the critical bands and should work very well, particularly if the center frequencies could be adjusted more finely. One-third octave matches low frequency critical bands, and should be sufficient at higher frequencies, particularly if a parametric equalizer is available to tackle areas requiring better resolution. The ideal equalizer might have 25 times the resolution to match our psychophysical resolution, but at greater expense and with marginal improvement for a good sound system.

3. FILTER RESPONSE AND BAND INTERACTIONS

The discrete equalizer filters are probably always based on second-order bandpass filters. Higher order filters have been shown to have so much phase shift that they are impractical for use in equalizers [19]. An examination of the analog second-order bandpass filter helps in understanding the equalizer characteristics, whether it is implemented as an analog circuit or in DSP. This filter has a magnitude response that can be quite sharp at its peak, but which asymptotically approaches a 6.02 dB per octave slope farther away, as shown in Figure 2, along with the phase response. This gives it the potential for significant effect over a wide frequency range. Its phase response is zero degrees at center, and approaches 90 degrees at low frequencies and –90 degrees at high frequencies. Its Nyquist plot traces a circular path in the complex plane as shown in

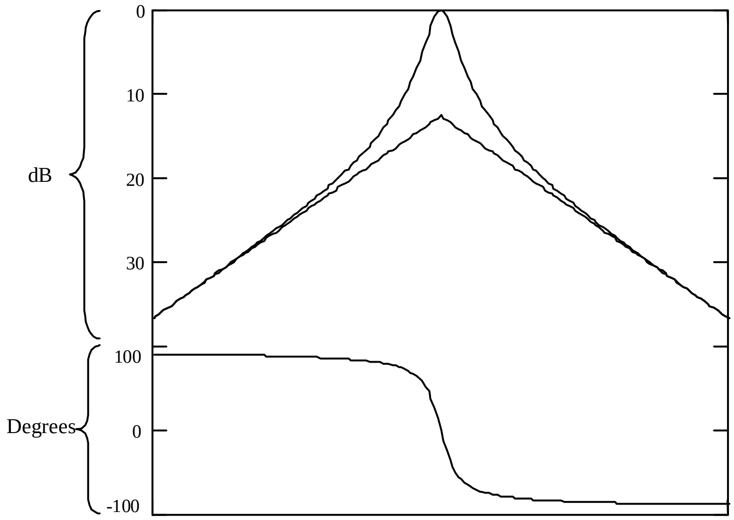


Figure 2: Bandpass Magnitude with the 6 dB/octave Asymptotes and Phase Below

Figure 3, with some analysis of this in Appendix I. The magnitude and phase are given by the length and angle of the vector from the origin to a point on the path.

In equalizer designs the bandpass may be summed proportionally with unity gain:

$$G(s) = 1 + (k-1)B(s) = \frac{s^2 + \frac{ks}{Q} + 1}{s^2 + \frac{s}{Q} + 1}, B(s) = \frac{\frac{\mathrm{s}}{\mathrm{Q}}}{\mathrm{s}^2 + \frac{s}{Q} + 1}, k = 10^{0.05dB} \qquad (1)$$

where k is the desired peak gain and B(s) has a given Q (without loss of generality, the design frequency is one here). The result has unity gain except near the filter center frequency, where the gain increases to k, and is commonly called a presence or bell filter, with resulting magnitude and phase shown in Figure 4, with Nyquist plot in Figure 3. The symmetry on the frequency axis of the bandpass is maintained.

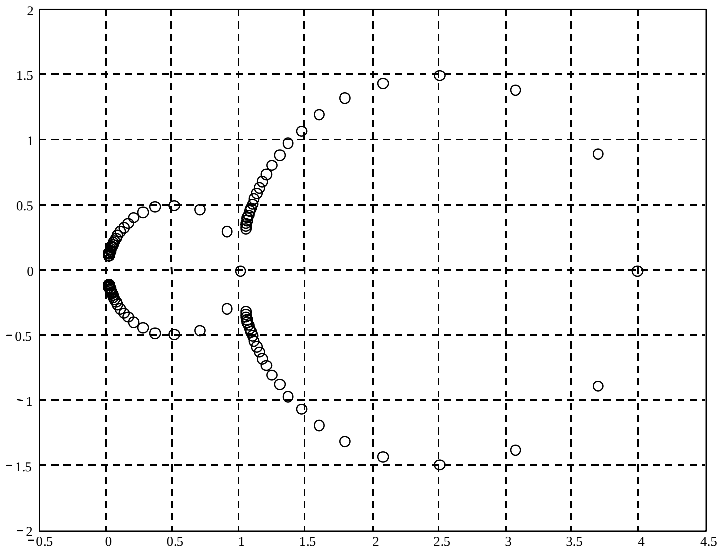


Figure 3: Nyquist Plot of Bandpass and 12 dB Presence Filters Plotted at One-Sixth Octave Intervals

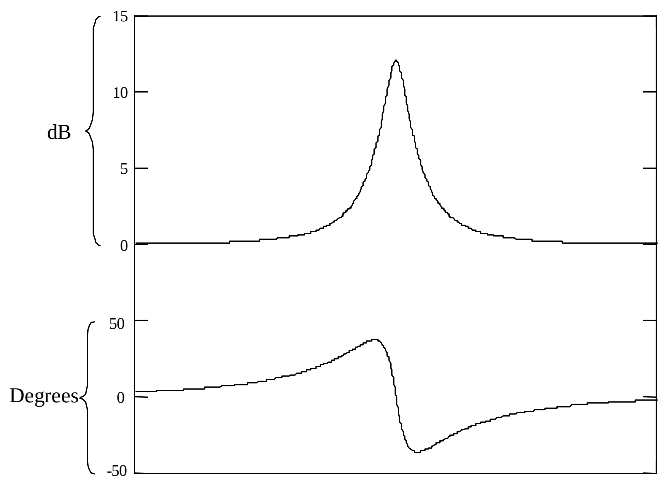


Figure 4: Presence Magnitude and Phase

A cut filter can be designed by using subtraction:

$$N(s) = 1 - \left(1 - \frac{1}{k}\right) \frac{\frac{s}{Q'}}{s^2 + \frac{s}{Q'} + 1} = \frac{s^2 + \frac{1}{k}\frac{s}{Q'} + 1}{s^2 + \frac{s}{Q'} + 1} \qquad (2)$$

The minimum level is $\frac{1}{k}$, however this will result in a somewhat narrower response width compared to the boost configuration as shown in Figure 5. For a symmetrical cut you simply use the reciprocal transfer function:

$$\frac{1}{1 + (k-1)B(s)} \qquad (3)$$

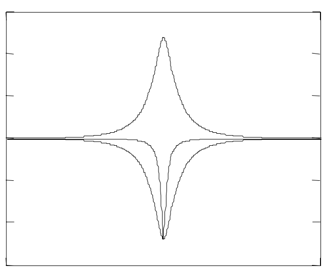


Figure 5: Boost and Symmetrical and Subtracted Cut

For the single filter case it turns out that subtraction with $Q' = \frac{Q}{k}$ also yields a symmetrical cut (see Appendix II).

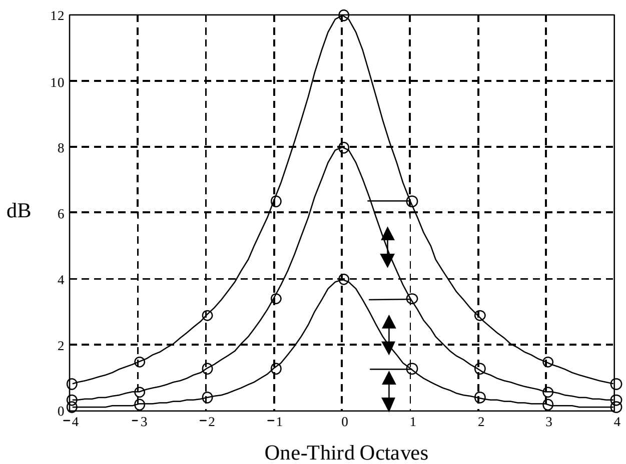
It is well known that any stable minimum phase transfer function can be inverted to yield another stable and minimum phase function. Whether the design is digital or analog, the condition for minimum phase zeros matches the condition for stable poles. Inversion is simply the reciprocal of the transfer function, and zeros and poles exchange places. This can be done in the analog domain by using feedback, or in the digital domain by swapping the filter coefficients and re-scaling. So as long as a filter is minimum phase, its symmetrical counterpart is given by the reciprocal transfer function.

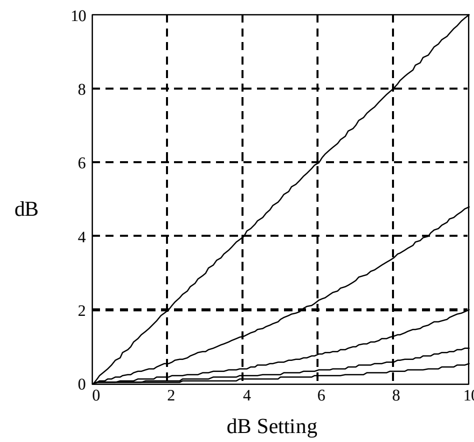
Analog equalizers and their digital counterparts are generally based on minimum phase filters. It's been shown that the various popular ways of combining multiple second order filters preserve the minimum phase property [21].

The boost and subtracted-cut presence filters have bandwidths which are obviously different, although they incorporate a bandpass with unchanging Q. A clarification of the definition of bandwidth is in order since many definitions are possible. For a bandpass filter, the bandwidth is specified by convention as the difference in frequencies that result in –3 dB magnitude response. Once this filter is combined in different ways with unity gain, the door is opened to specify bandwidth in different ways, such as the bandwidth of the underlying bandpass, the frequencies where the deviation from flat is +/-3 dB, or as points midway between 0 dB and the peak or valley (notice that if a peak is less than 3 dB, there are no -3 dB points). While these are all legitimate, one can alter the Q to achieve any of these bandwidths. Digital filters have progressively more warping near the Nyquist frequency, but can adjusted to compensate for this, either by modifying the Q or by a more sophisticated method [10,22]. For simplicity one can define the bandwidth as the bandwidth of the underlying filter, and then the presence filter bandwidth, referenced to the filter peak/valley, will approach the bandpass specification as the amount of boost/cut is increased.

Studying the presence filter magnitude response curve for different settings, one can see that while the center of the curve is naturally at the level of the setting, magnitude response at any distance from center is not a linear function of the setting. Figure 6 shows a family of response curves, and Figure 7 shows the magnitude response at nearby frequencies spaced one-third octave apart versus band setting. This magnitude function can be seen to be somewhat nonlinear:

$$20\log\left|H(\omega)\right| = 10\log\left[\frac{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2 + \frac{k^2\omega^2}{\omega_o{}^2Q^2}}{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2 + \frac{\omega^2}{\omega_o{}^2Q^2}}\right] = 10\log\left[\frac{\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2 + \frac{k^2}{Q^2}}{\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2 + \frac{1}{Q^2}}\right] \quad (4)$$





Figures 6 and 7: Presence Filter Response Skirt Nonlinearity at One-Third Octave from Band Center

Going from one band to thirty complicates the picture. One approach is to form a set of presence filters and cascade them. The complex responses multiply, so the total log magnitude and phase shift are simply the sum of the individual presence filter magnitudes and phase shifts. For two filters and a given signal frequency:

$$H_1(\omega) = M_1(\omega)e^{j\phi_1(\omega)},\ H_2(\omega) = M_2(\omega)e^{j\phi_2(\omega)} \quad (5)$$

$$H(\omega) = H_1(\omega)H_2(\omega) = M(\omega)e^{j\phi(\omega)} \rightarrow$$
$$\log M(\omega) = \log[M_1(\omega)] + \log[M_2(\omega)],\ \phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \quad (6)$$

Another is the parallel topology: to proportionally sum the bandpass outputs before summing with unity gain. In parallel, the result is the complex sum of the individual responses. The phase shift of the filters has the effect of cancelling much of the interband interference, while also increasing the response ripple a little (compared to cascaded filters) for a series of bands set to a nonzero level [19]. The only cancellation in the cascaded filters is within each filter, as the bandpass is summed with unity. Figure 1 shows composite responses for parallel, cascaded, and interpolated constant-Q (a combination of parallel and cascade).

Discrete filter fixed frequency equalizers fall into two categories, proportional-Q and constant-Q, which refer to the Q of the underlying bandpass filter. A proportional-Q equalizer has filters and band level controls tightly coupled together such that adjusting a control changes the filter Q. The Q can increase drastically with setting, yielding very broad response for small settings and sharp response for large ones. Although popular and effective, this is not at all predictable for the inexperienced user, being far from the settings in general. In the interest of true response, this paper focuses on equalizers with Q that doesn't vary radically.

Pioneering constant-Q equalizers represented a huge advance in the quest for a response that matched the settings [19]. The frequency resolution with moderate settings was enormously improved, and the combined response of adjacent bands, while it exceeded the settings, was much more accurate than proportional-Q designs. Parallel and cascade topologies have been combined to yield combined responses with relative flatness and accuracy. Parallel combination of alternate bands, then cascaded, is used to provide interpolated constant-Q equalization, where adjacent bands can be adjusted so that the response peak is moved between band centers [22].

While analog designs use multiple feedback loops, digital implementations have an additional constraint: that they can have no delay-free feedback loops. Otherwise a sample calculation could never complete. This prevents a digital bandpass filter with a delay-free path from simply being placed in a feedback loop, as is done in the analog domain for symmetrical cut. Equivalent transfer functions may be found in theory, and quite easily for single filters. Individual presence cut filters can be used with the cascaded topology, as long as the filters are very low-noise.

4. CORRECTION METHODS

One way to deal with interband interference is to remove it as needed using additional filters. This has been shown to be quite effective, although it requires more processing power [24].

One can also adjust the existing filters to yield the desired response, which is what one naturally does in practice. This can be automated and works well, but has limitations. To start with, it is difficult to make the response maximum at one band and minimum at the next. The width of the filter skirts require one to compensate drastically, and the required filter settings can become huge, as shown in Figure 8. In fact, for cascaded filters and a given fixed Q, if one attempts to adjust two adjacent filters to yield a given dB response (+N, -N) at band centers, there is a limit to the value of N, because the required settings become arbitrarily large (see Figure 9, Appendix III). This problem becomes more acute as the filter Q is decreased. Figure 10 shows how the inter-band ripple in the response increases as the filter Q is increased.

It seems that it would help to vary both the amplitude and bandwidth, and possibly also the center frequency, making the Q higher where it's needed for sharp transitions while keeping it lower otherwise to reduce ripple. But in general this is a substantial nonlinear optimization problem, and doesn't readily lend itself to real-time solution.

Various algorithms can be applied to these optimization problems [23]. There are simple algorithms such as steepest descent, and more sophisticated algorithms with faster convergence. A human being might simply look at the difference between the composite response and the settings, and iteratively adjust the settings by maybe half the error. This descent technique was applied automatically some time ago [25], and works well.

In general, optimization methods can perform the best, and can produce true response with any equalizer topology, given effective methods and high performance hardware. Otherwise they will perform slowly and with some caveats. It's possible for a method to converge unpredictably slowly, or to converge on a local minimum, and in that case the results may be unpredictable. The error criterion won't necessarily be appropriate for audio, resulting for example in a small

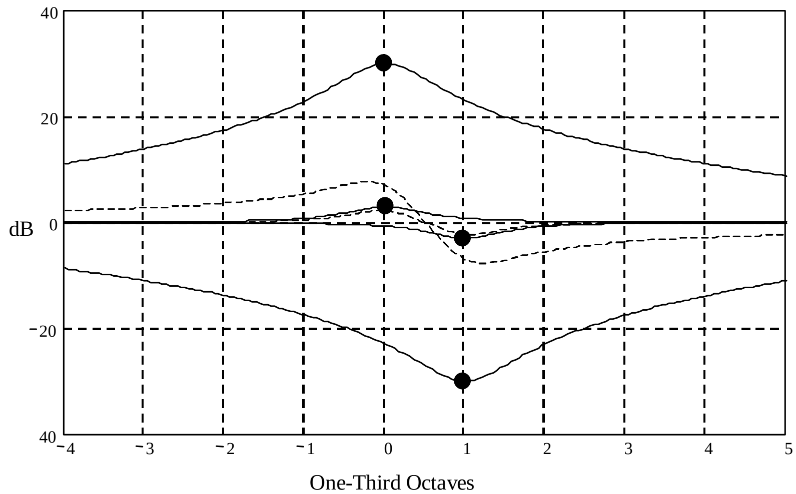


Figure 8: Abrupt Transitions Require Large Adjacent Band Settings

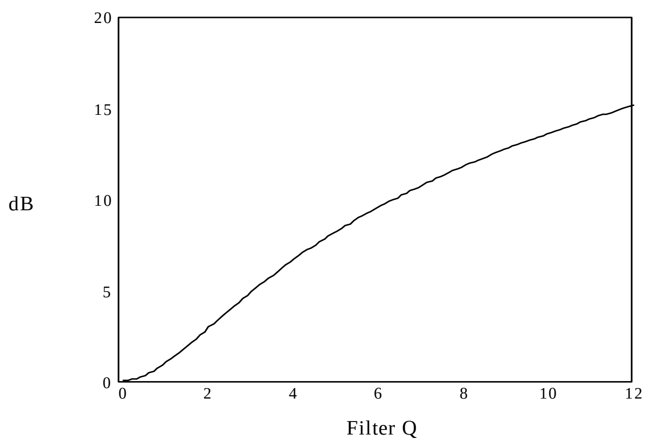


Figure 9: Maximum Response Half-Transition Between Adjacent Bands Versus Filter Q

Figure 10: Inter-band Ripple Increases with Filter Q of 2,3,4, and 5.

error over a large frequency range, instead of allowing a moderate error over a smaller range.

Since for cascaded filters the magnitude responses of the filters sum (in dB), and since the effect of a filter on nearby frequency response points is nearly linear over a moderate range, a linear approximation can be used [26]. A linear approximation can also be applied to other topologies. Given a fixed linear system (with an invertible system matrix) its inverse can be calculated once and applied to the settings dynamically to produce the internal filter settings. This results in the desired response at band centers, and it works well over a moderate range of settings. Beyond this, nonlinearity effects become significant.

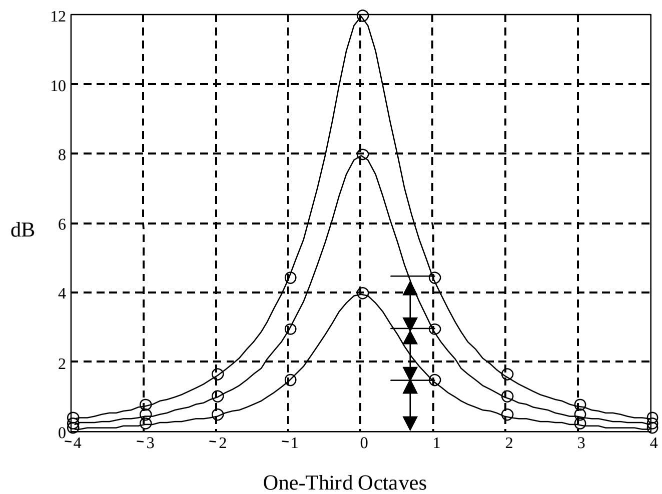
Since the most interband interference is from the two adjacent bands, a linear approximation would be more effective if those response points could be made a linear function of setting (by symmetry the magnitude at the left and right adjacent band frequencies are equal). This linear relationship can be produced by adjusting the filter Q as a function of setting such that adjacent band response is a linear function (in dB) of the setting[1]. Figure 11 shows a family of response curves, and Figure 12 shows the magnitude response at nearby frequencies spaced one-third octave apart versus band setting (compare to Figures 6 and 7). A closed form equation for the function Q(dB) is derived for this in Appendix IV.

It turns out that the response at more distant band frequencies is made more nearly linear, and is small enough that the nonlinearity is insignificant. The Q is highest for larger settings, yielding a magnitude response with excellent resolution with low ripple. If desired, the Q may be adjusted to optimize the linearity at several frequencies, but when this was done by the author, the result was found to be very close to the closed form solution. Ironically, a mild form of proportional-Q has been employed to achieve true response.

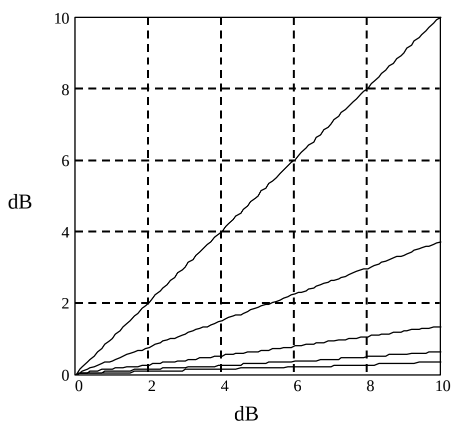
[1] This application of variable Q is covered by a patent application. Contact Rane Corporation for more details.

In theory this is fine, but there are remaining issues. When adjacent bands are set very differently the method will tend to produce overshoots just beyond band centers. Also, in this case the resultant filter settings may become larger than desired. These two issues may be handled effectively by means of small constraints on the settings (applied transparently), and this only slightly compromises the resolution.

Analog filters have been assumed here for simplicity, while digital filters may be more appropriate in practice. In this case, the response warping needs to be considered for high frequency bands.



Figures 11 and 12: Response Linearized at One-Third Octave from Band Centers



5. CONCLUSIONS

Although better performance is practical, equalizers designed using discrete filters have provided good functionality at low cost. Their main drawback has been the mismatch between settings and actual response, a situation which has been improved using creative series and parallel topologies. True response can be approximated to different degrees by many other techniques which digital technology has made more practical. One of these uses cascaded filters and a particular function which sets the filter Q based on boost/cut setting, thereby linearizing the response at neighboring bands, and allowing an excellent match. As technology progresses, large FIR filters or optimization techniques may provide rapid, precision control at low cost. Whatever the method, true response is a worthwhile objective.

6. APPENDIX I - DERIVATION AND COMPLEX RESPONSE OF THE SECOND ORDER BANDPASS

The second-order bandpass with $\omega_o = 1$ may be constructed from the first-order lowpass using the transformation (7) [20].

$$s = \frac{p^2 + 1}{p},\ p > 0 \qquad (7)$$

Substituting into a first-order lowpass with given cutoff, yields the bandpass (8)

$$\frac{1}{1+\frac{s}{\omega_b}} = \frac{1}{1+\frac{1}{\omega_b p}\left(p^2+1\right)} = \frac{\omega_b p}{p^2+\omega_b p+1} = \frac{\frac{p}{Q}}{p^2+\frac{p}{Q}+1},\ \mathrm{Q} = \frac{1}{\omega_\mathrm{b}} \qquad (8)$$

The cutoff frequency of the lowpass becomes the bandwidth of the bandpass.

It may occur to the reader that this second-order bandpass transfer function might not be unique. If instead one starts with a general transfer function which is a ratio of polynomials of degree two or less, and which has zero gain for both zero and infinite frequency, the denominator must have degree two and the numerator degree one. Add the requirement of a peak gain of one, it is easy to show that the form must be:

$$\frac{as}{s^2+as+1} \qquad (9)$$

Finally, add the condition that the magnitude response be 0.5 at $\omega = \omega_1$, which specifies a –3 dB bandwidth, and a little algebra yields

$$a = \omega_1 - \omega_1^{-1},\ \omega_1 > 1 \qquad (10)$$

The bandwidth is geometrically symmetrical about $\omega_o = 1$, resulting in:

$$\frac{\frac{s}{Q}}{s^2+\frac{s}{Q}+1},\ Q = \frac{1}{\omega_1-\omega_2},\ \omega_2 = \omega_1^{-1} \qquad (11)$$

So given the conditions above, this transfer function is unique and so is the transformation from first-order. The transformation can be evaluated alone for $p = j\omega, \omega > 0$:

$$s(\omega) = \left.\frac{p^2+1}{p}\right|_{p=j\omega} = p + p^{-1}\Big|_{p=j\omega} = j\left(\omega - \omega^{-1}\right) \tag{12}$$

This maps positive ω (p on the positive j axis) back to s on the entire j axis, but with the characteristic

$$s\left(\omega^{-1}\right) = -s(\omega) \tag{13}$$

which maps $s(1) = 0$, $\omega > 1$ to the positive j axis, and $0 < \omega < 1$ to the negative j axis. Plotted on a logarithmic frequency scale, $s(\omega)$ is antisymmetric with respect to $\omega = 1$.

The complex response of the first-order lowpass can be further analyzed, both for its direct use with positive frequency, and for bandpass use which includes negative s. The lowpass may be split into constant and allpass portions:

$\frac{1}{1+s} = \frac{1}{2}\left[1 + \frac{1-s}{1+s}\right]$, where the allpass portion $\left.\frac{1-s}{1+s}\right|_{s=j\omega}$ has a complex response that traces the unit circle, and can be shown to have phase shift $\theta = -2\tan^{-1}\omega$.

For positive frequency, the lowpass transfer function has a complex response that traces a semicircle with negative imaginary part. The bandpass transformation maps positive frequency onto the entire imaginary axis, and the complex response of the bandpass traces an entire circle (the point –1 needs special care).

A presence filter scales this path and shifts it right by one, as shown in Figure 3.

7. APPENDIX II - EQUIVALENCE OF SYMMETRICAL NOTCH AND SUBTRACTED NOTCH WITH MODIFIED Q

A presence filter is designed using proportional amounts of a bandpass summed with unity gain. For simplicity, the design angular frequency is one. The bandpass is:

$$B(s) = \frac{\frac{s}{Q}}{s^2 + \frac{s}{Q} + 1} \tag{14}$$

The resulting transfer function G is given by:

$$G(s) = 1 + (k-1)B(s) = \frac{s^2 + \frac{ks}{Q} + 1}{s^2 + \frac{s}{Q} + 1},\ k = 10^{0.05dB} \tag{15}$$

A symmetrical cut has the reciprocal transfer function:

$$H(s) = \frac{s^2 + \frac{s}{Q} + 1}{s^2 + \frac{ks}{Q} + 1} \tag{16}$$

Both the symmetrical cut and subtracted notch filters will require a minimum gain at the design frequency, and for equal but opposite dB values the required gain is (1/k). The Q for the subtracted notch case won't be assumed to be the same as before:

$$N(s) = 1 - \left(1 - \frac{1}{k}\right)\frac{\frac{s}{Q'}}{s^2 + \frac{s}{Q'} + 1}$$
$$= \frac{s^2 + \frac{1}{k}\frac{s}{Q'} + 1}{s^2 + \frac{s}{Q'} + 1} \tag{17}$$

Finally, comparing (16) and (17), one can see that they are equal if the Q is given by (18).

$$Q' = \frac{Q}{k} \tag{18}$$

8. APPENDIX III - MAXIMUM ALTERNATE BOOST/CUT ON ADJACENT BANDS ACKNOWLEDGEMENTS

The transfer function and squared magnitude of a bandpass filter proportionally summed with unity (presence filter) are given by

$$H(\omega) = 1 + (k-1)\frac{\frac{j\omega}{\omega_o Q}}{-\left(\frac{\omega}{\omega_o}\right)^2 + \frac{j\omega}{\omega_o Q} + 1} = \frac{-\left(\frac{\omega}{\omega_o}\right)^2 + \frac{jk\omega}{\omega_o Q} + 1}{-\left(\frac{\omega}{\omega_o}\right)^2 + \frac{j\omega}{\omega_o Q} + 1} \tag{19}$$

$$\left|H(\omega)\right|^2 = \frac{\left[1 - \left(\frac{\omega}{\omega_o}\right)^2\right]^2 + \frac{k^2\omega^2}{\omega_o{}^2 Q^2}}{\left[1 - \left(\frac{\omega}{\omega_o}\right)^2\right]^2 + \frac{\omega^2}{\omega_o{}^2 Q^2}} \tag{20}$$

Now for H_1, H_2 having center frequencies $\omega_o, \alpha\omega_o$, and with H_1, H_2 set to boost and cut respectively, in equal amounts, the result is (21):

$$\left|H_1(\omega)H_2(\omega)\right|^2 = \left[\frac{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\frac{k^2\omega^2}{\omega_o{}^2Q^2}}{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\frac{\omega^2}{\omega_o{}^2Q^2}}\right]\left[\frac{\left[1-\left(\frac{\omega}{\alpha\omega_o}\right)^2\right]^2+\frac{\omega^2}{\alpha^2\omega_o{}^2Q^2}}{\left[1-\left(\frac{\omega}{\alpha\omega_o}\right)^2\right]^2+\frac{k^2\omega^2}{\alpha^2\omega_o{}^2Q^2}}\right]$$

$$= \left[\frac{k^{-2}\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\frac{\omega^2}{\omega_o{}^2Q^2}}{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]^2+\frac{\omega^2}{\omega_o{}^2Q^2}}\right]\left[\frac{\left[1-\left(\frac{\omega}{\alpha\omega_o}\right)^2\right]^2+\frac{\omega^2}{\alpha^2\omega_o{}^2Q^2}}{k^{-2}\left[1-\left(\frac{\omega}{\alpha\omega_o}\right)^2\right]^2+\frac{\omega^2}{\alpha^2\omega_o{}^2Q^2}}\right] \quad (21)$$

The taking the limit as k approaches infinity, the squared-magnitude is:

$$M(\omega)=\lim_{k\to\infty}\left|H_1(\omega)H_2(\omega)\right|^2=\alpha^2\,\frac{\left[1-\left(\frac{\omega}{\alpha\omega_o}\right)^2\right]^2+\frac{\omega^2}{\alpha^2\omega_o{}^2Q^2}}{\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right]+\frac{\omega^2}{\omega_o{}^2Q^2}} \quad (22)$$

The maximum possible magnitude at ω_o is given by $M(\omega_o)$, which reduces to:

$$M(\omega_o)=Q^2\left(\alpha-\alpha^{-1}\right)^2+1 \quad (23)$$

A one-third octave equalizer has $\alpha = 2^{\frac{1}{3}}$, so then $M(\omega_o)\approx 0.21736Q^2+1$

This being the magnitude squared, the maximum in dB for a Q of 4 is only about 6.51 dB. Although the peak of the magnitude curve lies just to the side of the band center, it is only slightly larger. By symmetry, the magnitude at the other band center is equal but with opposite sign.

9. APPENDIX IV - CLOSED FORM SOLUTION FOR Q(K)

Given a presence filter, the filter Q can be adjusted as a function of setting, Q(k), such that the magnitude response in decibels at a preselected frequency varies linearly with the setting in decibels.

The frequency ω_1 can be considered the ratio of the preselected frequency to the filter design frequency, and the complex response function H is given by:

$$H(\omega_1,k)=1+(k-1)\frac{\frac{j\omega_1}{Q(k)}}{-\omega_1{}^2+\frac{j\omega_1}{Q(k)}+1}=\frac{1-\omega_1{}^2+\frac{jk\omega_1}{Q(k)}}{1-\omega_1{}^2+\frac{j\omega_1}{Q(k)}} \quad (24)$$

The linear dB constraint with proportionality constant C can be written:

$$20\log_{10}|H(\omega_1,k)|=20C\log_{10}k$$
$$=20\log_{10}(k^C) \quad (25)$$

Taking the squared magnitude yields (26):

$$|H(\omega_1,k)|^2=\frac{\left(1-\omega_1{}^2\right)^2+k^2\omega_1{}^2Q^{-2}(k)}{\left(1-\omega_1{}^2\right)^2+\omega_1{}^2Q^{-2}(k)} \quad (26)$$

Combining (25) and (26) results in:

$$\frac{\left(1-\omega_1{}^2\right)^2+k^2\omega_1{}^2Q^{-2}(k)}{\left(1-\omega_1{}^2\right)^2+\omega_1{}^2Q^{-2}(k)}=k^{2C} \quad (27)$$

Solving for the function Q yields:

$$Q(k)=\frac{\omega_1}{\omega_1{}^2-1}\sqrt{\frac{k^2-k^{2C}}{k^{2C}-1}},\omega_1>1 \quad (28)$$

The constant C is still undetermined, but is found by taking the log of both sides of (27), and choosing a particular value of k_o and $Q(k_o)$:

$$C=0.5\frac{\log\left[\frac{\left(1-\omega_1{}^2\right)^2+k_o{}^2\omega_1{}^2Q^{-2}(k_o)}{\left(1-\omega_1{}^2\right)^2+\omega_1{}^2Q^{-2}(k_o)}\right]}{\log k_o} \quad (29)$$

Any reasonable values can be used for k_o and $Q(k_o)$, but k_o should near the midrange of normal filter band boost adjustment. Larger $Q(k_o)$ will increase response ripple and lower $Q(k_o)$ will increase the amount of correction required by this method.

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